MODELS OF SPATIAL COMPETITION: A CRITICAL REVIEW

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Abstract

This critical review focuses on the development of spatial competition models *à la* Hotelling in which the location choice of firms plays a major role. We start by quantifying the research in this field by using bibliometric tools. Thereafter, this study identifies the main research paths within spatial competition modelling. Specifically, the type of strategy (Bertrand *vs.* Cournot competition), the assumptions that are made in respect to the market (linear vs. circular), production costs, transportation costs, the number of firms and the type of information (complete vs. incomplete) and their effects on the location equilibria are also discussed.

1. INTRODUCTION

Spatial economics is "*concerned with the allocation of scarce resources over space and the location of economic activity*" (Duranton, 2008: 1). It may therefore be related to a very broad set of questions, as most economic questions involve space and location issues. However, according to Duranton (2008), the main focus of spatial economics is the location choice of the economic agents.

In order to explain how agents choose to locate in certain places, specific modelling problems arise because of the difficulty of inserting location in the framework in a realistic way.

The starting point is the neoclassical paradigm, which assumes perfect competition and constant returns to scale. Accordingly, Debreu (1959) suggests that spatial economics is all about adding a spatial dimension to the goods and agents, meaning that every commodity and agent has different characteristics because they are located in different places, while there are transportation costs of commodities between different locations. In this framework, economic activities will be evenly distributed across a homogeneous space.

However, Starrett (1974) came up with a particular model where the locations are homogenous. Each location, as long as the production and consumption of goods are perfectly divisible and transportation is costly, will satisfy its own needs, reducing its transportation costs to zero, operating as an autarchy. Therefore, the equilibrium results failed to mirror the reality as there is no trade between different locations in the economy: every agent would maximize its utility by interacting only in its location. This finding gave rise to the Spatial Impossibility Theorem, which states that models of competitive equilibrium never involve transportation of commodities, which is counter-factual.

In order to explain the location choices of economic agents and the agglomeration of agents in certain locations, one must relax the core assumptions of the competitive framework. According to Fujita and Thisse (2002), three alternatives emerged and received huge attention in the literature: the assumption of heterogeneity of locations, in which there is an uneven distribution of resources, as in comparative advantage models (e.g. Ricardo, 1963 [1821]; Hecksher-Ohlin, 1991 [1919]) or in pioneering static location models (e.g. Von Thünen, 1966 [1826]; Weber, 1929 [1909]); the externality models, in which the economic activity endogenously generates spillovers that motivate the agglomeration of the agents (e.g. Marshall, 1920; Henderson 1974); and the assumption of imperfect markets, implying that agents have to interact with each other, with location being an important variable, as in spatial competition models (e.g. Hotelling, 1929) or in the monopolistic competition approach (e.g. Lösch, 1954 [1940]; Krugman, 1991).

This review will focus on the development of spatial competition models *à la* Hotelling. Specifically, the main purpose is to study models in which the location choice by the firms plays a major role, instead of those models in which, regardless of the spatial nature of price competition, the location of the firms is fixed.

This topic is extremely appealing, firstly because it mixes Game Theory tools with Regional and Urban Economics in order to explain firms' locations; secondly, because it offers some interesting insights into Industrial Organization, because of firms' strategic interaction and behaviour; and finally, because of the huge literature in this research field and the recent insights gained regarding asymmetric information and its application to this subject. As a whole, this topic makes a very solid contribution to micro-economic science.

In section 2, the roots of spatial competition are reviewed. In section 3, along with a bibliometric approach to the papers in this area, some of the most important developments in the field are presented, with the focus directed at the optimal location decision. Section 4 presents concluding remarks.

2. SPATIAL COMPETITION – THE ROOTS

Spatial competition is mainly concerned with the locational interdependence among economic agents under the constraints of imperfect competition. According to Smith (1981), the first major contribution to studying interdependence among firms was by Fetter (1924), who constructed the law of market areas. According to Fetter, consumers compare the prices in both firms and the freight costs needed to buy that product before making their choice and the locations of consumers who are indifferent about buying at either location defines the market boundary of those firms. Some of Fetter's ideas influenced the work of most location theorists in the 1930s, but the most influential paper was that of Hotelling (1929).[1]

The model that he developed was in fact one of the most significant historical landmarks in the development of Location Theory. In his model there exists a city represented by a line segment, where a uniformly distributed continuum of consumers has to buy a homogenous good in order to survive. Consumers have to pay transportation costs when buying the good, which is to be bought from one of the two firms existing in the city. Within this framework, firms simultaneously choose their locations and afterwards set their prices in order to maximize their profits.

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[1] One can notice that the Fetter's law of market areas is present in Hotelling's framework, but Fetter overlooked the issue of the optimal location or even the optimal price decision of the firms and was more concerned about modelling the demand behaviour of the market.

Hotelling was actually more intent on proving the existence of a stable equilibrium in duopoly markets than developing a spatial framework. According to him, the main feature of the paper was the elimination of discontinuities in the demand of each firm, *i.e.* small changes in price would only capture part of the demand existing in the market, which would solve the Bertrand (1883) paradox, in which small changes in price would capture the whole market for one of the firms, leading the firms to an (unrealistic) equilibrium situation with no profits.

Moreover, Hotelling did not think of his framework as a location model, despite mentioning transportation costs. He introduced "distance" between firms as a way of modelling differentiation between the goods produced in each firm, with the goods being homogenous except for the location where they were produced, which is a similar concept of location introduced later by Debreu (1959).

However, in the second part of the paper, Hotelling introduced the following question: given the location of a firm, which is the location for the other firm that maximizes its own profits? This question attracted scientific attention to this framework, which was extended in numerous ways in order to answer many different questions within, for instance, location theory (as will be shown later), game theory, industrial organization, social welfare and even mathematical issues such as the existence of equilibrium.

In a quite different approach, Chamberlin (1950) introduces the concept of monopolistic competition. This approach arises because of product differentiation, in which firms may combine characteristics of being both in a monopoly and in pure competition, as they possess a somewhat unique product in a competitive market. Product differentiation may refer to many characteristics of the product, including its location. This "middle point" between pure competition and monopoly has new implications for the behaviour of the firms when it comes to maximizing their profits. The parallel with the Hotelling framework is evident, as the "linear city" is meant to represent product differentiation throughout the market under study.

This review follows the framework of Hotelling, as the subsequent publications around this framework are more concerned with the agents' location behaviour than the developments of Chamberlin, which are used more as a building block for product differentiation or than the framework of Fetter, which has been relatively forgotten.

3. DEVELOPMENTS IN SPATIAL COMPETITION MODELLING *À LA* HOTELLING: A CRITICAL REVIEW

3.1 A bibliometric exercise on research in spatial competition

Before proceeding to the analysis of the main contributions in spatial competition modelling that focus on the location decisions of firms, a numerical study is conducted in order to better understand the temporal development of the field. The analysis begins in 1979, the year that d'Aspremont *et al.* (1979) published what can now be considered a classic paper in the field, and ends in 2011.

The search engine used was Scopus and only articles in the subject area of "Social Sciences & Humanities" were considered. Document type was filtered to only include peer-reviewed articles and exclude comments, rejoinders, book reviews and *corrigendas*. The database was constructed using the keywords "spatial competition" or "Hotelling" that were sought in the articles' title, keywords and abstract[2]. Finally, in order to develop a clear description of spatial competition modeling we have excluded any record that is not related with this field, by direct inspection of each article's title and abstract.[3] As a result, the database includes a total number of 352 journal articles published since 1979. Our intention is to give an idea of the development of the field, without intending it to be completely exhaustive.

By analyzing the distribution through time, we can see a gradual increase in publications, suggesting an increase in the field (Figure 1). However, in relative terms, compared with the total number of peer-reviewed articles in Scopus – Social Sciences and Humanities, that is not the case, with an irregular trend in the importance of spatial competition over time being observed (Figure 2). [4]

(Figure 1 here)

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[2] As an alternative, we searched for the keywords "spatial competition" OR "Hotelling" in whole texts of papers, obtaining a total of 4,037 articles. However, most results were not directly related to the topic under study, and therefore, we chose to search only in titles, abstracts and keywords. Additionally, we also searched for the words 'spatial competition' in articles' titles, abstract and keywords (305 records) and in the whole text (1,230 records), as well as other possibilities ("spatial OR spatially" AND "competition OR competitive" OR "Hotelling" OR 'product differentiation'"; spatial AND competition), and adopted the best option as far as proximity to our subject was concerned.

[3] We had this necessity because Hotelling was also known for a statistical test, a famous rule in the field of exhaustible resources and for the "Hotelling's lemma" in microeconomic theory. Therefore, we excluded these articles to obtain a better assessment of research in spatial competition.

[4] It should be said that Scopus database covers a large set of journals after 1996, but has some limitations in the period before, which might justify the increasing pattern shown in Figure 1, as well as the absence of d'Aspremont *et al.* (1979) in the searched records
(http://www.info.sciverse.com/UserFiles/sciverse_scopus_content_coverage_0.pdf).

This evidence of the importance of the field of spatial competition is not surprising to anyone who is familiar with the literature discussed in the remaining of this section. In fact, spatial competition was a hot topic in the eighties and nineties, when a huge modelling effort was devoted to examining the effects of changing every Hotelling assumption on the subsequent equilibrium conditions.

(Figure 2 here)

With respect to the authors' efforts regarding the spatial competition modelling, information about the most relevant researchers is displayed in Table 1. Jacques-François Thisse and Stefano Colombo are the authors more articles in this research field, with 8 publications each, immediately followed by Debashis Pal, Noriaki Matsushima and Ralph Braid. In addition, when we take into consideration the average number of citations *per* paper, we may conclude that Jacques-François Thisse is one of the most prominent researchers on this topic, together with Nicholas Economides and Takatoshi Tabuchi. Additionally, information about authors' geographic affiliation (Figure 3) reveals the importance of European authors' research into spatial competition.

(Table 1 here)

(Figure 3 here)

In order to assess the quality of the research in spatial competition modelling, a selection of the most frequent journals in this field has been undertaken (Table 2). As expected, the vast majority are journals specialized in *Regional and Urban Economics*, besides other journals dealing with *Industrial Organization* or *Public Economics*. However, it is not only specialized journals that are interested in spatial competition, as more general ones also contain articles in this field, with *Economics Letters* and *European Economic Review* amongst those with the most publications in this area of research. Regarding the impact factor of these journals, we can see that 67.85% of the articles published in journals containing 3 or more articles on this particular topic have a 5-year impact factor higher than 1, meaning that a significant number of publications in the field have at least a moderate impact.

(Table 2 here)

To sum up, spatial competition models have seen regular growth in terms of the number of publications. Furthermore, most of those models have been published in journals with at least "moderate" impact.

After this brief bibliometric overview of the research into spatial competition modelling, this paper critically reviews the main models for each of the four research paths that we have identified after the work of Hotelling (1929). These paths are ordered according to the greatest frequency of publication, as exemplified in figure 4. The first group is Bertrand competition, which immediately follows Hotelling's (1929) model and shows the highest number of publications; secondly comes Cournot competition, differing from Hotelling's spatial-price competition, focusing on quantity competition in the second stage; in third place are non-linear markets such as circular or triangular markets, diverging from Hotelling's linear city; more recently, models of incomplete information between players have appeared, which extend Hotelling's complete information model.[5]

Throughout the remainder of the review, the focus is directed on the papers related to the location behaviour of the agents, rather than their pricing or quantity behaviour. This means that other important articles of "spatial competition *à la* Hotelling", possibly included in the bibliometric search undertaken earlier, are not reviewed.

<center>(Figure 4 here, in a separate, horizontally-oriented page)</center>

3.2 Bertrand Competition

3.2.1 Mill Pricing

The Hotelling model was an ideal basis for examining the behaviour of firms when it comes to their price and location decisions because it allows for easy understanding and an appealing logic, and also because of its usefulness in studying firms' interactions. The Hotelling model is based on the following assumptions: two firms are the players in a two-stage location-price game, in which at the first stage, firms must choose their location on a linear and bounded city and at the second stage compete on prices. The good sold by the firms is homogenous except for the location they have chosen in the first stage. Demand is perfectly inelastic; *i.e.,* consumers in that city must buy one unit of the good, while incurring a linear transportation cost when travelling to one of the firms. In the second-stage, firms compete in a mill price setting, *i.e.,* they choose a price for their good, bearing in mind that each consumer takes into account the price plus the transportation costs when deciding from which firm to buy the good. In the mill

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[5] It should be said that in the bibliometric approach, it is impossible to separate the papers between these different research paths because of the difficulty of finding keywords that are able to do so. For example, comparisons between Cournot and Bertrand competition are very frequent in papers of both research paths. As a result, no single keyword can reliably identify whether a paper contained in the search belongs to a particular research path.

price setting, a Nash equilibrium in the price stage is defined when both firms simultaneously choose prices (given their previous choice of locations) that maximize their profits, given the price set by the other firm.

With these assumptions, Hotelling concluded that firms would agglomerate at the centre of a linear city, thereby laying the foundations for the "Principle of Minimum Differentiation", so called by Boulding (1966). This principle was undisputed and was used as a starting point for research, with its conclusions being studied and extended into many branches of research. However, almost half a century later some scientists started to question this principle, mainly by using the Hotelling model with some different, usually more realistic, assumptions. The most important conclusion is the one drawn from d'Aspremont et al. (1979), which introduced quadratic transportation costs[6]. The introduction of this feature removed the discontinuities verified in the profit and demand functions, which was a problem in the Hotelling model since there were no Nash price equilibrium solutions for all possible locations of the firms. The location decision for the firms in the presence of quadratic transportation costs is to locate at the extremes of the market (principle of maximum differentiation). Firms wish to differentiate more and more in order to relax price competition and thus obtain larger profits.

Following this paper, the majority of the models abandoned the linear transportation costs assumption, except for the cases where scientists were once again testing the cost functions, such as Gabszewicz and Thisse (1986) and Anderson (1988), who tested a transportation cost function with a linear and a quadratic component. They proved that in some cases there is no price equilibrium for fixed symmetric locations and that in most cases no location-price equilibrium exists in the two-stage location game.

The assumption for the bearer of the transportation costs was changed in Anderson and Thisse (1988), Anderson et al. (1989) and Hamilton et al. (1991), to name but three instances. Hamilton et al. (1991) introduce a model where consumers are allowed to bargain between the two firms, which makes the firms choose the socially optimum locations, 0.25 and 0.75. This bargaining in the model is only possible because firms do not observe the consumers' locations in the city. The other authors reached no specific conclusions regarding location patterns: Anderson and Thisse (1988) and Anderson et al. (1989) focus more on the existence of equilibrium than the location of the firms.

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[6] Quadratic transportation costs are a realistic assumption when we are thinking of transportation costs different from the physical costs, for instance, consumer tastes.

In Hotelling's model, firms were interacting in a linear and bounded market, with only one differentiating dimension, and selling homogenous goods. Demand was assumed to be perfectly inelastic, meaning that consumers will always buy one unit of the good, whatever the price (if there is no reservation price).

While the linear and bounded market assumptions seem not to be too binding, the others seem quite unreasonable in terms of reality, but are easily understandable. The analysis of the equilibrium of the two-stage game with more than one dimension or with elastic demand proved to be a hard obstacle to overcome, while finding a way to quantify heterogeneity of the goods was not obvious.

In an ambitious paper, Irmen and Thisse (1998) extend the Hotelling problem to an n-dimensional market where consumers may weight each dimension differently. They conclude that when a characteristic is sufficiently strong, the situation in which the firms fully differentiate in one characteristic and locate in the centre for all the others is a global equilibrium for the usual two-stage game. Therefore, "Hotelling was almost right", in the sense that firms apply the principle of minimum differentiation except for the most important characteristic.

Some authors addressed the assumption of homogenous goods by introducing heterogeneity into the model. Three different approaches appeared in the literature: De Palma *et al.* (1985), Anderson *et al.* (1989) and Ben-Akiva *et al.* (1989).

De Palma *et al.* (1985) only changed the homogeneity of the goods and concluded that when prices are fixed and equal for both firms, agglomeration at the centre occurs and the profits for the firms grow with the degree of heterogeneity of the products (when the degree equals zero, we have the Hotelling case). In the two-staged game, agglomeration equilibrium may occur, but only if the degree of heterogeneity is high enough. Anderson *et al.* (1989) tested different price schemes for a fixed location by comparing consumer and producer surplus in those cases, but since locations are fixed no conclusions can be drawn regarding location theory. Ben-Akiva *et al.* (1989) introduced a second dimension to the problem by introducing brands which are intended to model heterogeneity. When firms play for location and price simultaneously with exogenous brands, agglomeration equilibrium at the centre of the city occurs if the heterogeneity in consumer tastes is not too small.

Anderson and Engers (1994) solved the two-stage location-price game for more than two firms and assumed an elastic demand. The conclusion for the case of two firms is that if the demand is perfectly inelastic (Hotelling) or sufficiently inelastic, such firms will still prefer to agglomerate at the centre.

However, the nature of this game is different from that of Hotelling, as pricing in the second-stage is defined by a social planner.

The main feature of Anderson *et al.* (1997) was to change the density of the consumers to a symmetric log-concave function. The conclusion is that if the density function is too concave, asymmetric equilibrium appears in the location decision. Furthermore, if the density function is concentrated more at the centre, that does not always lead to closer equilibrium locations. Transportation costs make no difference to the equilibrium location. Moreover, with this specification of the density function, there is excess differentiation in the product compared to the social optimum.

Hotelling considered the case of only two firms in a two-staged game, deciding first their location and then prices simultaneously with pure strategies. However, the characteristics of this game have also been changed to address different issues or to search for a better overall realistic framework.

Dasgupta and Maskin (1986) proved the existence of mixed-strategy equilibrium for the pricing sub-game for all possible locations of the firms, paving the way for Osborne and Pitchik (1987), who discovered that when mixed strategies are allowed only at the second stage, using only pure strategies in the first stage, the symmetric location where firms are located at 0.27 and 0.73 is an equilibrium. This equilibrium is near the social optimum, which are the location of firms that minimize the total transportation costs of the population. However, the transportation costs per unit distance in this model were set as a constant equal to one.

Anderson (1988), as previously mentioned, concluded that there is no pure strategy perfect equilibrium for most cases when the transportation cost has a linear and a quadratic component. However, allowing for mixed strategies at the price stage, the game becomes well defined, but if the transportation function is not convex enough, symmetric location equilibria must involve mixed strategies in pricing.

Two very different approaches follow: Ben-Akiva *et al.* (1989) introduced exogenous brands into the firms: a brand "is given" to all firms in the market, which have simultaneously to choose location and price. However, consumers face linear transportation costs when travelling to a location and quadratic costs when "travelling" to a different brand in relation to their tastes. In this setting, agglomeration equilibrium exists at the centre of the city only if heterogeneity in tastes is considerable enough. Moreover, the smaller the number of firms in the market, the more likely it is that the agglomeration will result in an equilibrium. This result is very similar to the one found in Irmen and Thisse (1998), since if

firms are able to differentiate on brands, they have the incentive to choose the central location because price competition is already softened due to product differentiation.

Friedman and Thisse (1993) introduce a game in which location is played in the first stage, and then there is a repeated game in which players keep choosing prices for n periods. As the game is repeated, firms play a trigger strategy regarding prices. The equilibrium for this game is agglomeration at the centre of the city, with collusion for both firms only at the price-stage.

Boyer *et al.* (1994) study the case of sequential location decisions within a uniform delivered price setting. In this game with three stages, one firm chooses its location first, followed by the other firm, after which both firms enter into price competition. With transportation costs equal to one and equal marginal costs, firms choose to locate at 2/5 and 4/5, respectively. The same framework, but with the mill pricing setting, was studied by Boyer *et al.* (2003a). If firms have the same marginal costs, the results are the same as those of d'Aspremont *et al.* (1979). However, if one firm has an advantage in its marginal costs, it starts to move progressively to the centre, while the firm with the higher marginal costs always chooses the opposing extreme of the market.

Lambertini (2002) builds a model where two firms enter a market sequentially, *à la* Hotelling, but take the lag between the entries of both firms as a variable. The main conclusion is that the longer the second firm takes to enter, the closer the first firm will locate to the centre, while the second firm will always choose one of the extremes of the market. The first firm prefers to locate in the centre because there it can set a higher price and still capture the entire market. This happens due to the existence of a reservation price for consumers.

More recent extensions have been identified. Liang and Mai (2006) added vertical subcontracting to the model; Aguirre and Espiñosa (2004) introduced consumer arbitrage; Matsumura and Matsushima (2004) introduced heterogeneous firms with endogenous production costs; Lai and Tsai (2004) introduced zoning regulation and Degryse *et al.* (2009) introduced different transportation costs for each consumer.

3.2.2 Spatial Price Discrimination Setting

Another frequent way of treating price competition in the linear city model of Hotelling is by introducing the so-called spatial price discrimination. In this setting, firms, instead of fixing a single price in their store, are allowed to set a price for each location in the city. This price will no longer be the price at the store, but is the delivered price, *i.e.* including the transportation costs, which are now incurred by the firms. This setting allows firms, when they are monopolists, to fix the maximum price possible in each

location, given that each consumer still buys the good. When there is more than one firm, a Nash equilibrium in this price sub-game occurs when all the firms participating in the game do not wish to change their delivered price, given the delivered price set by the other firms, for every location in the city. This setting was introduced by Hoover (1937) and Lerner and Singer (1937), while analyzing the location results of Hotelling and using Hotelling's other assumptions. Hoover (1937) was more cautious in deriving a location result, since it depended too much on the type of industry considered; while Lerner and Singer (1937) argue that the equilibrium locations of a finite number of discriminating firms on a unit interval are the same as the socially optimal locations. Greenhut and Greenhut (1975) studied the profits and prices of firms with different exogenous firm locations. Although not directly studying the two-stage location-price game *à la* Hotelling, this paper had a significant influence in the spatial price discrimination literature. However, in spite of the relevance of these three papers, their results are not entirely related to the two stage location-price game that is the main object of this review.

Moreover, within this setting, the usual focus of researchers is not the two-stage location-price game. Most of the papers present models in which the location of both firms is fixed, and so the focus is the profit and price results of the firms, as well as social welfare for the agents in the linear city. In this subsection, the focus is on the papers that contribute to explaining the location choice of firms.

A starting point is Hurter and Lederer (1985). The authors state that the location of the plants in a city that minimizes social costs is an equilibrium of the game. The reason is that every plant benefits largely from having consumers that live close by, allowing them to price discriminate effectively among those consumers without the "intrusion" of other plants.

An interesting exception is Anderson and de Palma (1988), who study the case where the products are assumed to be heterogeneous regarding consumer tastes. They conclude that, when the degree of heterogeneity is zero, the model is equivalent to Hurter and Lederer (1985)'s model and the location result is the social optimum. However, as the degree of heterogeneity rises, both firms move away from the city centre. But a further increase in the heterogeneity brings the firms closer, and after a given threshold, agglomeration at the centre is an equilibrium of the two-stage location-price game. This non-linear behaviour happens because of two opposite effects: when the degree of heterogeneity increases, the concept of market areas becomes blurred, *i.e.* a consumer may prefer the firm located on the left, while another consumer located to the left of the first consumer may prefer the firm to its right. This makes the firms more competitive, and so they prefer to differentiate more in order to lessen price competition.

However, as the degree of heterogeneity grows sufficiently, firms gain more monopoly power, as changes in prices become less important in defining each firm's demand, which causes firms to locate closer to each other. This last effect is similar to the one observed by De Palma *et al.* (1985) and Ben-Akiva *et al.* (1989).

The paper of Anderson *et al.* (1989) provides a comparison between the profits and prices in different pricing schemes for the firms when products are exogenously heterogeneous in the eyes of the consumers. Although the locations are fixed, one can conclude that when in a duopoly, the profits for the firms and total social surplus are both higher in a mill pricing than in a price discrimination setting for any value of products' heterogeneity.

Although they focused more on finding the equilibrium conditions in a circular market setting (a disc), Lederer and Hurter (1986) proved that it is impossible for two identical firms that are price discriminating to be located at the same point in the market, since this leads to zero profits for both firms. In the same line of research, MacLeod *et al.* (1988) allow the firms to choose the number of stores to build in a linear city. After concluding that there is a Nash Equilibrium at the price sub-stage for every possible location of the plants, the location that minimizes social costs still remains an equilibrium of the game. In terms of the number of firms entering the market, the authors are inconclusive, stating that: "In addition, we find that equilibrium may generate the socially optimal level of product variety, but may also produce more or less product variety than is socially optimal" (MacLeod *et al.,* 1988: 444).

In a short paper, Gupta (1992) examines whether firms still locate at the socially optimum places in the case of sequential entry. The paper states that, in the case of two firms, the first mover will locate closer to the centre (0.4) while the second mover has to settle at a greater distance from the central location (0.8).

Braid (2008), after a good summary of the results in the literature, models a two-stage location-price game in which consumers have an exogenous preference (other than location) over the goods of the firms. The author concludes that firms choose to locate in the socially optimum locations for the model.

To conclude this subsection, we can state that the location results for the game when firms are allowed to price discriminate against the consumers, according to literature, tend to be around the socially optimum values. This seems to happen because of two effects: on the one hand, each firm is interested in locating as far as possible from its opponents, allowing a better price discrimination against the consumers that are, in a sense, exclusive to the firm; on the other hand, what keeps the firms from settling at the extremes

of the market is that they are responsible for paying the transportation costs of the good. Therefore, firms want to locate in a place that minimizes their transportation costs when transporting the goods to the potential demand. These two effects lead to the straightforward conclusion that in a duopoly in which the linear city model is symmetric in all its characteristics, *i.e.* in which neither side or firm has an advantage, firms share the market evenly and locate in the middle of their market areas, which coincides with the socially optimum result that minimizes the transportation costs of the economy.

3.3 Cournot Competition

This review will now deal with the two-staged location game in which firms compete à la Cournot (in quantities), instead of competing à la Bertrand (using prices), at the second stage. Hotelling originally created this game with price competition and maybe that is why this assumption is the one most frequently made in the literature.

The assumption of competition in quantities is usually less realistic than competition in prices when we think about competition among firms. The price of a good is an important determinant of the demand for it in most cases, while the quantities placed in a market seem to be a more indirect determinant of demand. However, in modelling non-spatial duopoly cases, the Bertrand (1883) model produces less realistic results than the Cournot (1897 [1838]) model.

In some industries, however, competition in quantities is a better assumption than competition in prices: the Cournot assumption is more appropriate for markets where quantity is less flexible than price at each market point (Anderson and Neven, 1991; Pal and Sarkar, 2002), and also when there are significant lags between the production decision and the price setting (Hamilton *et al.*, 1994). It is not a surprise, then, that some authors have decided to analyze these kinds of location games.

It should be stressed that Cournot competition shows some significant differences relative to the Bertrand competition case. In the two-stage location-quantity game, firms select their location simultaneously and then choose the quantities supplied. However, at the second stage, instead of setting a quantity for the whole market and waiting for the consumers to travel to their store (as in the Bertrand case), each firm chooses to supply a quantity for each location in the city (similar to the spatial price discrimination setting, applied to quantities), which implies that the combination of quantities chosen by each firm in each location determines the price of the good in each location. Thus, a Nash equilibrium is defined at this second stage when for all locations in the city, all the firms set a quantity such that there is not a

single firm that wishes to change its quantity delivered, given the quantities delivered by other firms (*i.e.* there must be a Nash equilibrium in all the locations of the city).

As the reader may have noticed, the agents that pay transportation costs within this framework are the firms, as they have to take the good to each location in the city. This framework can be better understood if we think that firms compete in a typical Cournot setting in every location of the city, with their "marginal costs" equal to the price of the good plus the cost of delivering the good to the chosen location. The profits of the firms will be the sum of the resulting profits in all locations of the city.

In terms of results, this different framework has new implications. In Bertrand competition with a mill-price setting, firms have their own market areas based on the existence of an indifferent consumer. In quantity competition, firms compete in every location of the city in a typical Cournot setting. Therefore, instead of having a "market area", both firms may sell their homogenous good everywhere in the city, which seems to provide a more realistic result.

Additionally, the assumption of inelastic demand must be dropped, since competition in quantities would result in corner solutions in which the price would be infinite, somewhat analogous to the zero-profit condition in Bertrand competition (Hamilton *et al.*, 1989).

Greenhut and Greenhut (1975) adapted the setting of spatial price discrimination, allowing for more than one firm competing in the market. Although not directly based on the Hotelling framework, firms select quantities when interacting with each other. This paper derived the profile of the delivered price schedule, paving the way for future studies into Cournot Competition.

The baseline case used in this section will be that of Hamilton *et al.* (1989), which compared the case of price and quantity competition. The authors conclude that in the framework of quantity competition, for all values in which there exists a solution, firms will always agglomerate in the central location of the city. This is in contrast with the case of price competition, in which firms never agglomerate for any feasible range of values for transportation costs, given exactly the same assumptions.

Anderson and Neven (1991) extend these results by studying the equilibrium conditions of this two-staged location game. Ensuring that the reservation price is high enough such that in all locations every consumer buys from both firms, they conclude that when the demand is linear and transportation costs are convex, there is a unique equilibrium in the game, where both firms locate at the centre of the market. Furthermore, for any changes in the demand or cost transportation functions, any location equilibrium in this game must involve symmetric locations between firms.

Later, Chamorro-Rivas (2000a) relaxed the assumption of high reservation prices and found that for lower reservation prices, the agglomeration equilibrium at the centre ceases to be unique, although it is still an equilibrium. For even lower reservation prices, Benassi *et al.* (2007) find that the central agglomeration location is no longer an equilibrium result. The unique equilibrium found is a dispersed symmetric equilibrium. Therefore, agglomeration does not hold when the reservation price (transportation costs) is too low (or high).

Hamilton *et al.* (1994) examine the two-staged game of location and quantities with Cournot competition where consumers pay the linear transportation costs. In this framework, there is no pure strategy equilibrium in quantities for all possible locations of the two firms (see Hamilton *et al.* (1994), p. 913, for a very intuitive graphical explanation). However, considering only the case for symmetric firm locations, the authors solve the two-stage game and conclude that firms locate very near to the centre, where low values for transportation costs pertain, even if at the second stage mixed strategies are played.

The following three papers, in line with the Bertrand competition strand, changed some assumptions regarding firms' and consumers' conditions for operating in the market.

Mayer (2000) introduces the assumption of different production costs throughout the city, meaning that the location of the firms also matters in relation to the cost structure for the production of the goods. The main result is that if the global convexity of the production cost distribution holds, there is an agglomeration equilibrium result between the minimum cost location and the centre. Depending on the cost distribution of the city, firms face a trade-off between the demand effect and the diminution of the marginal cost of production. However, they may still agglomerate even if it is not at the central location.

Gupta *et al.* (1997) change the distribution of consumers in the city using a consumer density function, in a similar way to Anderson *et al.* (1997), in the case of price competition. They conclude that in the case of two firms, non-agglomeration cannot occur if the population density is sufficiently "thick" for all points of the city. Also, the agglomeration equilibrium found is unique.

Shimizu (2002) introduced product differentiation into the Hamilton *et al.* (1989) framework. However, the main location result does not change: the central agglomeration equilibrium is unique for any degree of differentiation of the product.

Extensions also appeared in the case of competition within n firms. Anderson and Neven (1991) concluded that all firms agglomerate at the centre, given linear demand and linear transportation costs,

while Gupta *et al.* (1997) proved that agglomeration is the unique equilibrium if the non-uniform consumer density is not too "thin" along the linear city.

Pal and Sarkar (2002) introduced the interesting case whereby two firms compete by having more than one store, *i.e.* they can choose more than one location in the city. The main conclusion is that if the two firms have the same number of stores and the demand is considerable in relation to transportation costs, both firms choose their monopoly locations, thus partially agglomerating in the city. The results for the case where firms have a different number of stores vary significantly depending on the numbers involved.

Regarding Cournot competition, more recently Chen and Lai (2008), in a similar way to Lai and Tsai (2004), extend the literature by analyzing the effects of zoning regulations on the optimal decisions of firms; Wang and Chen (2008) introduce the hiring of workers by firms and analyze the equilibrium conditions with wage bargaining.

We can see that fewer assumptions from the Hotelling model in the location-quantity game were changed throughout time compared with the location-price game. This is one proof that the literature on price competition is more developed and that it is the result of the high attention that location theorists have paid to this kind of competition, seeking to solve the Bertrand paradox.

To sum up regarding quantity competition, one can say that in these models *à la* Hotelling, the conclusions are similar to those relating to Industrial Organization: Cournot competition has less realistic assumptions, such as the delivered price setting and the competition in quantities itself, which is less realistic than competition in prices; however, the results are more realistic, as the agglomeration result may be obtained more easily, and it is a fact that firms sell everywhere in the city, in contrast with the less realistic result of a "market area" for each firm.

3.4 Non-linear markets

One of the lines of research that followed Hotelling (1929) abandons the assumption of a linear market while remaining in the two-stage location-price framework.

Why should one work on circular markets? First of all, it is interesting to analyze the results for the location of firms, given that there are no extremes in the market. One can see that in the circular market, no location is *a priori* better than another, which is not the case in linear market models (Gupta *et al.*, 2004). Secondly, there are some markets that may be better represented by a circular market, for instance,

time-dependent markets, such as television companies who must choose time slots to broadcast their programmes (Gupta *et al.*, 2004).

Salop (1979) varied the Hotelling framework by assuming that consumers are located on a circle rather than on a line segment, although his paper is not the first to assume a circular city model (see Vickrey [1999 (1964)] or Eaton and Lipsey (1975) for an early reference). The choice of this city specification is due to allowing "*the "corner" difficulties of the original Hotelling model to be ignored*" (Salop 1979: 142). This paper does not undertake an analysis of the two-stage location game, because it takes location as given. However, it is important as a starting point for all the subsequent two-staged game analysis in circular markets.

In the context of spatial price discrimination, Lederer and Hurter (1986) conclude that when firms have identical marginal costs and transportation rates, the agglomeration result cannot be an equilibrium. Moreover, when firms are not identical, the equilibrium involves both firms being located on the opposite side of the diameter of the circle when the market is given by a disc.

In a short paper, Pal (1998) introduced the circular market into the two-stage location game in order to prove that Cournot competition does not yield spatial agglomeration in all situations. He concluded that, in equilibrium, two (or more) firms will locate equidistantly from each other on the city circle, which is a maximum differentiation result. Matsushima (2001) extended the conclusions to the case of n firms and proved the existence of partial agglomeration equilibrium, that is, half of the firms agglomerate at a point and the other half agglomerate at the diametrically opposite point of the circular city.

Chamorro-Rivas (2000b) extended the analysis for two firms that can have more than one plant. In the case of two firms and two plants, the conclusion is that in equilibrium, the plants will be located in each quarter of the market, with each firm setting its plants at diametrically opposite points.

Gupta *et al.* (2004) take an important step in the study of circular markets, by identifying multiple equilibrium locations for a given number of firms, in which the findings of Pal (1998) and Matsushima (2001) are included. The highlight of the results is the existence of a huge number of equilibrium locations, though none of them involves agglomeration of all firms at the same point. An interesting result is that in the case of an even number of firms, all equilibrium situations yield equal profits and equal consumer surpluses.

Matsumura *et al.* (2005) extend the previous framework by assuming nonlinear transportation costs. However, the paper considers the existence of four isolated markets in the city rather than a continuum of

consumers. The main objective was to assess which equilibrium (Pal, 1998 *vs.* Matsushima, 2001) was the more robust, by checking its existence, given different configurations of the transportation cost function. It is shown that in the case of simultaneous entry, the location pattern identified by Pal is always an equilibrium, while the one identified by Matsushima only occurs if the transportation costs are not "too concave or too convex". In the case of sequential entry, the location pattern of Pal is the unique equilibrium if the transportation costs are non-linear. Therefore, dispersion equilibrium seems more robust than partial agglomeration equilibrium.

There are a number of variations of the Salop circular city model. For instance, Brueckner *et al.* (2002) distribute the firms and the skills of the workers in a circular city, adapting the framework to the study of labour markets; and Arakawa (2006) applies the framework to studying the location problem of shopping centres.

In short, the conclusions arising from the assumption of circular markets are quite different from those found in the previous sections of this review. The main differences are that while a unique equilibrium was easier to find in a linear market setting, multiple equilibria often arise in a circular market. Moreover, agglomeration of all firms in one location is never an equilibrium outcome in circular markets, in which at most partial agglomeration arises.

A different type of market is studied by Braid (1989), who examines the two-stage location-price game along intersecting roadways, and concludes that there is no equilibrium in the first stage of the game, for any number of firms. Another possibility is to consider triangular consumer densities in one dimensional models. Tabuchi and Thisse (1995) showed that if the model of d'Aspremont et al. (1979) is extended to allow firms to locate outside the unit interval, then the symmetric location pattern (-1/4, 5/4) is obtained, but if the model is further extended to allow a symmetric triangular consumer density with a peak in the middle of the unit interval, then there are two possible location patterns, both asymmetric: (-0.272, 0.680) and (0.320, 1.272). In a slightly different approach, Tsai and Lai (2005) assume a market in which consumers are located uniformly in three different lines that form a triangle. Firms are restricted to choose their location in only one of the lines, denoted as the "main street". However, similarly to Tabuchi and Thisse, firms are also allowed to locate outside of the interval that forms the "main street" line. For a symmetric triangle, the authors conclude that firms locate in the location pattern (-1/4, 5/4), similarly to the uniform distribution case and contrary to the findings of Tabuchi and Thisse for a symmetric triangle.

3.5 Incomplete Information

Now we shall turn to a more recent strand in the literature, dedicated to studying the location equilibrium of firms in cases where the agents do not have perfect information about the game. As is known, the assumption of perfect information is quite unrealistic, as firms usually do not know the precise cost structure of the other firms or even the tastes of the consumers regarding their product and other competitors' products. The literature on this subject will differ depending on the type of lack of information assumed.

In some of the following models, location is usually observed by all the firms and therefore it is used by the incumbent or by the first mover as a signal to the other firms of its cost structure or the quality of its good, previously determined by "nature", which may be defined as a signalling game (Macho-Stadler and Perez-Castrillo, 2001).

Boyer *et al.* (1994) study the case of sequential location decisions within a delivered price setting, where two firms choose their locations and afterwards their prices in a context of asymmetric production costs. Firm 1 first chooses its location having either equal or lower marginal costs with a given probability. Asymmetric information arises because firm 2 does not know the marginal costs of firm 1 before choosing its location. Therefore, location for firm 1 is used as a signalling mechanism for its cost structure.

When the difference between the marginal costs is low, a unique refined separating Perfect Bayesian equilibrium (PBE) exists, with firm 1's location being closer to the centre compared to the case of complete information when it has both low and high costs. However, when the difference between the efficiency of high and low cost firms becomes too marked, the only refined PBE is pooling, and the incumbent finds it more profitable to locate in the same place independently of its cost efficiency. Its position in the linear city will depend on the beliefs of firm 2 that firm 1 is a lower cost firm.

Later, Boyer *et al* (2003a) developed a similar model, but with a mill pricing setting. In this case, there is a unique separating equilibrium if firm 1's possible disadvantage is not great enough or if it's possible advantage is very significant, which implies that high-cost firm 1 locates at the extreme and the low cost firm moves progressively to the centre as its possible advantage is great, while firm 2 locates at the other extreme. If the relative advantage is not too big (for either of the sides), there is a unique pooling equilibrium at the extremes of the city for both firms.

In a similar case, Boyer *et al.* (2003b) study the case where there is an incumbent who might have a high or low marginal cost and an entrant who has to decide if it will enter the market. However, the entrant does not know the true cost of the incumbent, which allows the latter to use location as a signalling mechanism. Agglomeration equilibrium never occurs, for both delivered and mill price settings. This happens because in the pooling equilibrium, the incumbent chooses a central location, preventing the entry of the second firm, while for the separating equilibrium, whenever the incumbent chooses a location closer to the centre it is because it is a low-cost firm, thus pushing the entrant to the other extreme of the market.

The following models have problems of lack of information, but have a different modelling perspective, other than the signaling game explained by Macho-Stadler and Perez-Castrillo (2001).

Vettas and Christou (2005), allowing for vertical differentiation, study Hotelling's two-staged location game. Two firms know the existing quality difference between them, but do not know who has the better quality, which is a problem of lack of information. In the first stage they decide on their locations, while in the second they know the relationship between both qualities and so compete in prices.

If there is no quality difference between the firms, the results for the location game are the same as those shown by d'Aspremont *et al.* (1979). As the quality difference grows, firms tend to draw close to the centre. This mechanism occurs because firms compete in prices, which implies that the equilibrium prices when the firms are agglomerated are exactly the quality difference for the firm with the higher quality and zero for the other firm. Therefore, there is an incentive to agglomerate if this quality difference (keeping the transportation cost constant) improves, because the possible monopoly profits are very high in the case of a firm with better quality.

In a rather different setting, Aiura (2010) studies the equilibrium locations of three firms when location is decided sequentially among them. That is, the game has three stages, at the first stage of which firm 1 chooses its location and so on until all the three firms have chosen their locations. Prices are fixed, which implies that maximizing profits is equal to maximizing demand. The linear city is [-1, 1], but the consumers of the good are only located in [θ, $\theta+1$], with θ belonging to the interval between -1 and 0. The asymmetric information problem arises because firms do not know θ when choosing their locations. However, the subsequent firms can observe the demand of those that have already chosen their location, thus updating their beliefs about θ. Although this is not a classic problem of information presented in microeconomics, it is very similar to a signaling problem.

The Perfect Bayesian Equilibrium result is that firm 1 locates in the centre, firm 2 also locates in the centre and firm 3 unambiguously chooses to locate infinitesimally to the left or right of both firms. The rationale is very intuitive: firm 1 chooses the value that is expected to capture the maximum demand possible in the future. Firm 2 chooses the same as firm 1 in order not to provide firm 3 with any kind of information. Firm 3, since it does not know anything about the true location of θ, will randomly choose to capture one of the two sides of the market. Therefore, agglomeration equilibrium at the centre of the city occurs in this interesting case.

The following model by Valletti (2002) is a typical case of adverse selection. The consumer has private information before the purchase of the good and therefore the firm has to design different goods and prices for each type of consumer (Macho-Stadler and Perez-Castrillo, 2001).

Valletti (2002) builds a model where consumers are distributed within a linear city but there is also a vertical component, determined by the quality of the good. In each location, there are two types of consumers: the ones who prefer a high-quality product and those who prefer the low-quality one.

Therefore, the two-stage location game played by the duopoly firms is slightly different from the Hotelling location-price game. Firms in the first period choose their location but in the second period firms offer discriminatory contracts, as is usual in the case of principal-agent problems.

The conclusions regarding the locations in the two-stage location game depend on the ratio between the high-quality and the low-quality goods demanded by the consumers. However, firms' locations will always be close to the socially optimal levels for any value of this ratio. The main changes that different values for the ratio induce are in the distribution of the surpluses between the firms and the consumers.

To conclude, the agglomeration results in this literature seem to depend heavily on the type of asymmetric information assumed. Models without the standard specification of asymmetric information are able more easily to find conditions for agglomeration of the firms.

Within a different framework, Rusco and Walls (1999) develop an auction model, in which two firms located at the extremes of the market compete for the purchase of some good, which is randomly located somewhere within Hotelling's linear city. The game has two stages: in both stages, firms participate in an auction in order to acquire the good. The main feature of the model is that the firm that wins the first stage will have an expected lower utility in the second stage auction. The imperfect information issue arises because firms do not know where the second auction will take place, which will condition their

behaviour at the first stage, since if they lose the first auction they will have a relative advantage over their opponent in winning the second auction.

Although this approach does not reach any conclusions regarding the location of the firms, its interesting framework may be developed in order to explain the location behaviour of firms when participating in an auction.

4. CONCLUSION

After the appearance of the Hotelling (1929) model and the important findings of d'Aspremont *et al.* (1979), scientists had access to a simple and successful means of introducing a spatial component into the modelling behaviour of economic agents. This review has focused on the developments that were intended to explain the equilibrium locations of the firms, mainly when competing in a duopoly. However, many successful variants of this framework were used to justify spatial price discrimination and different market specificities, to furnish two examples.

In the 80s, this field became a hot topic for research. There are numerous applications of the Hotelling model, which mainly focus on changing the framework assumptions. The field developed significantly with the successful modelling experience of Hamilton *et al.* (1989), which allowed for competition in quantities.

More recently, Pal (1998) combined the circular framework of Salop (1979) in order to study the location decision of firms. In addition, the development of the asymmetric information framework in microeconomics and its successful adaptation to the context of spatial competition again led to the extension of the field. However, these last approaches did not receive similar attention.

After a brief look at the numerical exercise done in section 3.1, it would seem that most of the important features that justify the spatial behaviour of firms have already been explored. The future of the field depends on the researchers' capacity to find an (even more) interesting and innovative way of studying spatial competition. There is a high proportion of spatial competition models *à la* Bertrand or *à la* Cournot, compared with the most recent assumptions laid out in this review. In that sense, future researching efforts in spatial modelling should be made in the incomplete information setting.

Furthermore, researchers could intensify the relationship between spatial competition and Industrial Organization. For example, spatial competition may provide a more complete answer in relation to

vertical differentiation/integration of duopoly firms or to the R&D investment decisions by firms, in line with the seminal work of d'Aspremont and Jacquemin (1988).

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Figures and Tables

Figure 1 – Number of articles on Spatial Competition, 1979-2011
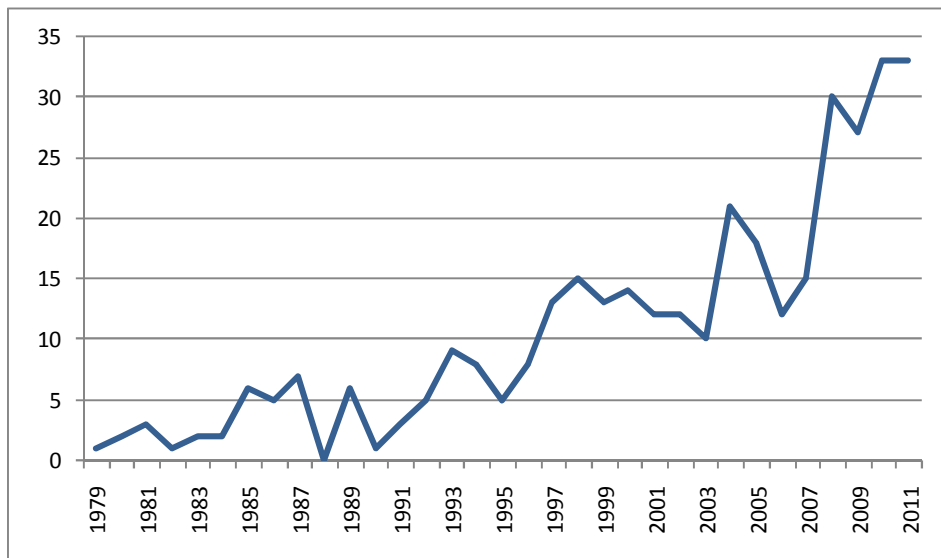
Figure 2 – Published articles on Spatial Competition (% of total Scopus), 1979-==2011==
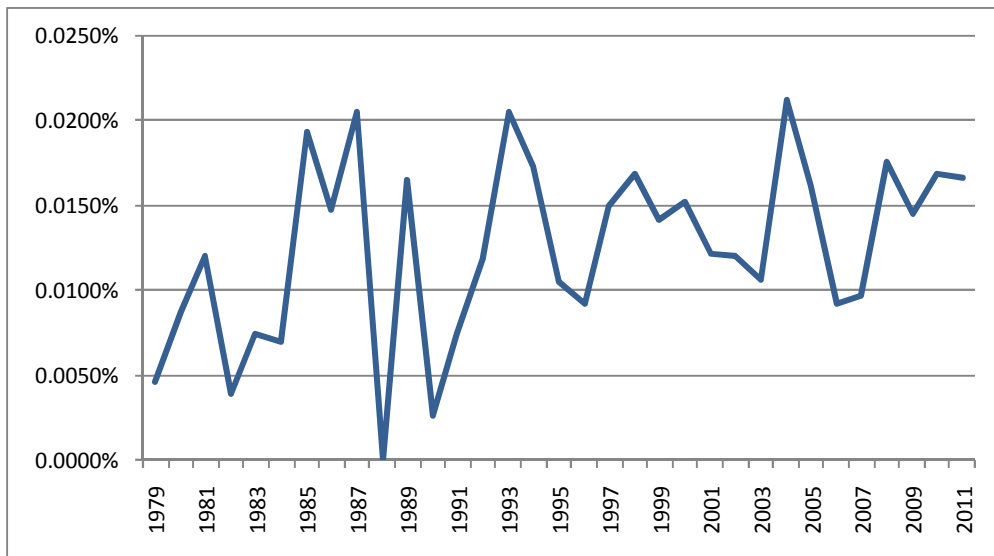
Table 1 – Top authors in Spatial Competition, 1979-2011

Author	Number of articles	Citations *per* article
Thisse, J.F.	8	28.38
Colombo, S.	8	1.75
Pal, D.	7	21.57
Matsushima, N.	7	11.14
Braid, R.M.	7	3.29
Gupta, B.	5	16.60
Matsumura, T.	5	10.60
Norman, G.	5	5.20
Economides, N.	4	30.00
Tabuchi, T.	4	29.00
Sarkar, J.	4	21.75
Hamoudi, H.	4	14.00
Lai, F.C.	4	11.75
Lambertini, L.	4	7.50
Straume, O.R.	4	7.00
Meagher, K.J.	4	4.00
Sanjo, Y.	4	3.25
Adams, J.	3	18.67
Yu, C.M.	3	15.67
Mai, C.C.	3	11.67
Brekke, K.R.	3	8.00
Rothschild, R.	3	7.33
Thill, J.C.	3	6.67
Zauner, K.G.	3	5.00
Sexton, R.J.	3	4.67
Georgantzis, N.	3	2.33
Chakrabarti, A.	3	1.33
Grofman, B.	3	1.33
Merrill, S.	3	1.33
Aiura, H.	3	0.67
Andaluz, J.	3	0.67
Evrenk, H.	3	0.67
Matsubayashi, N.	3	0.33

Figure 3 – Authors' affiliation, 1979-2011

North America — 34.30%
Europe — 46.86%
Asia — 14.98%
Others — 3.86%
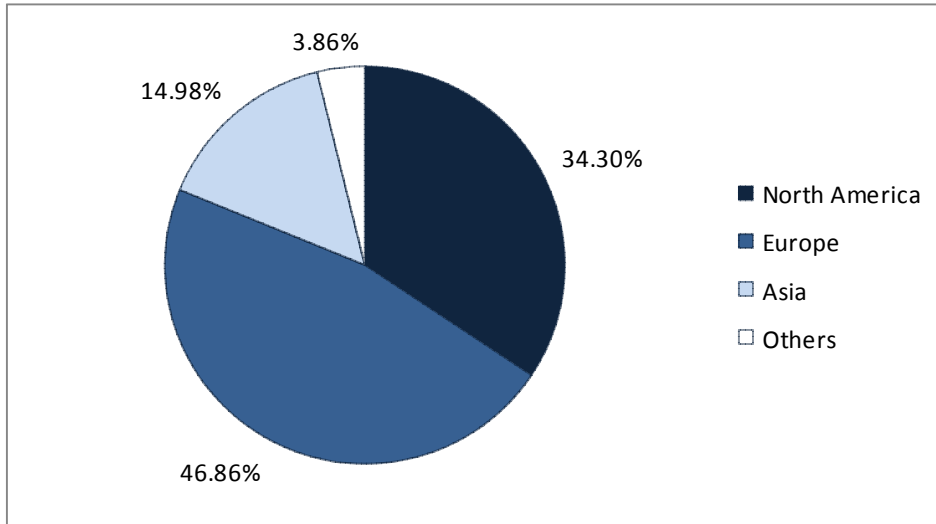
Table 2 – Top journals in Spatial Competition, 1979-<mark>2011</mark>

Journals	Number of articles	% of total Spatial Competition	5-year impact factor
Regional Science and Urban Economics	45	12.78%	1.612
Economics Letters	34	9.66%	0.627
International Journal of Industrial Organization	30	8.52%	1.247
Public Choice	12	3.41%	0.996
Papers in Regional Science	10	2.84%	1.638
Economics Bulletin	10	2.84%	N.C.
Journal of Economics Zeitschrift Fur Nationalokonomie	10	2.84%	N.C.
European Economic Review	9	2.56%	1.775
Journal of Regional Science	7	1.99%	1.391
Journal of Industrial Economics	7	1.99%	1.678
Journal of Urban Economics	7	1.99%	2.607
Social Choice and Welfare	6	1.70%	0.716
Journal of Economic Theory	6	1.70%	1.511
Economic Theory	5	1.42%	0.932
Games and Economic Behavior	5	1.42%	1.503
Journal of Economics and Management Strategy	5	1.42%	1.656
Research in Economics	5	1.42%	N.C.
Shanghai Jiaotong Daxue Xuebao Journal of Shanghai Jiaotong University	5	1.42%	N.C.
Annals of Operations Research	4	1.14%	1.223
European Journal of Operational Research	4	1.14%	2.512
B E Journal of Theoretical Economics *	3	0.85%	0.303
International Journal of Health Care Finance and Economics *	3	0.85%	0.415
Mathematical Social Sciences	3	0.85%	0.561
Applied Economics	3	0.85%	0.740
Annals of Regional Science	3	0.85%	0.984
Mathematical and Computer Modelling	3	0.85%	1.166
Marketing Science	3	0.85%	2.996
Management Science	3	0.85%	3.966

<mark>* 2010 impact factor</mark>

Figure 4 – Research paths in Spatial Competition

Hotelling (1929)
- Two firms
- Two-stage Location-Price game
- Linear and Bounded City
- Linear Transportation Costs
- Perfectly Inelastic Demand
- Homogeneous product
- Mill pricing
- Pure-strategies equilibria
- Complete Information

BERTRAND COMPETITION

Quadratic transportation costs
d'Aspremont et al. (1979)

Linear and Quadratic transportation costs
Gabszewicz and Thisse (1986)
Anderson (1988)

Market characteristics
Eaton and Lipsey (1975)
Braid (1989)
Irmen and Thisse (1998)
Anderson et al. (1997)

Delivered pricing
Lederer and Hurter (1986)
Anderson and Thisse (1988)
Anderson et al. (1989)
Hamilton et al. (1991)

Heterogeneous product
De Palma et al. (1985)
Anderson et al. (1989)
Ben-Akiva et al. (1989)

Elastic demand, "n" firms
Anderson and Engers (1994)

Mixed strategies
Dasgupta and Maskin (1986)
Osborne and Pitchik (1987)
Friedman and Thisse (1993)

Spatial Price Discrimination
Hoover (1937)
Hurter and Lederer (1985)
MacLeod et al. (1988)
Anderson and De Palma (1988)

COURNOT COMPETITION

Elastic demand and
Delivered pricing
Greenhut and Greenhut (1975)
Hamilton et al. (1989)
Anderson and Neven (1991)
Chamorro-Rivas (2000a)
Benassi et al. (2007)

Mill pricing
Hamilton et al. (1994)

"n" firms
Anderson and Neven (1991)
Gupta et al. (1997)

Heterogeneous product
Shimizu (2002)

NON-LINEAR MARKETS

Bertrand competition and
Circular markets
Salop (1979)

Triangle markets
Tsai and Lai (2005)

Cournot competition and Circular markets
Pal (1998)

with "n" firms
Matsushima (2001)
Gupta et al. (2004)

and Non-linear transportation costs
Matsumura et al. (2005)

INCOMPLETE INFORMATION

Production costs
Boyer et al. (1994)
Boyer et al (2003a)
Boyer et al. (2003b)

Quality
Tropeano (2001)
Vettas and Christou (2005)

Demand
Aiura (2010)
Valetti (2002)

Auctions
Rusco and Walls (1999)